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## THE THERMAL CLASSIFICATION OF LAKES\*

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Forel<sup>1</sup> introduced, for holomictic lakes of sufficient depth to exhibit thermal stratification, a threefold classification in which the following categories were recognized: *polar* lakes, never at any point over 4° C., inversely stratified under ice in winter and circulating in summer; *temperate* lakes, inversely stratified in winter, directly stratified in summer, and circulating twice a year at about 4° C.; and *tropical* lakes, always at every point above 4° C., circulating in winter and directly stratified in summer.

The terminology has been long considered inappropriate, as many of the finest examples of Forel's tropical type occur in western Scotland and in British Columbia, regions far outside the tropics, while, of the few polar lakes studied, more occur at high altitudes in temperate and tropical latitudes than within the Arctic Circle. It has also become apparent, since the magnificent work of Ruttner<sup>2</sup> in Indonesia, that the typical lake of the humid tropics at low altitudes circulates rarely, in an irregular manner, not at a specific season but when exceptional periods of cool weather permit sufficient heat loss at the surface to cause instability. This presumably happens at intervals of length greater than that of a year.

Recently one of us (H. L.) has had the opportunity to study lakes in equatorial latitudes but at great altitudes, in the Andes. Although at low altitudes in the humid tropics small temperature gradients can maintain stable stratifications, no stable stratification develops at the low temperatures of high altitudes, where the density difference per degree centigrade is very small. The lack of seasonal variation, that permits almost perennial stratification at low altitudes in equatorial latitudes, thus permits perennial circulation at high altitudes in the same latitudes.

We propose for these two types of equatorial lake the terms *oligomictic* and *polymictic*, respectively. For the three categories of Forel's classification the terms *cold monomictic*, *dimictic*, and *warm monomictic* have been used by one of us (G. E. H.) in a forthcoming extensive work on limnology.<sup>3</sup> The theoretical scheme (Fig. 1) may be completed by recognizing for the very rare perennially ice-covered

lakes of the antarctic and, under special conditions, of some high mountains (Löffler unpublished) a category of *amictic* lakes, perennially sealed off by ice from most of the annual seasonal variations in temperature.<sup>4</sup>

It is evident from the considerable body of work on Central Africa that in the dry

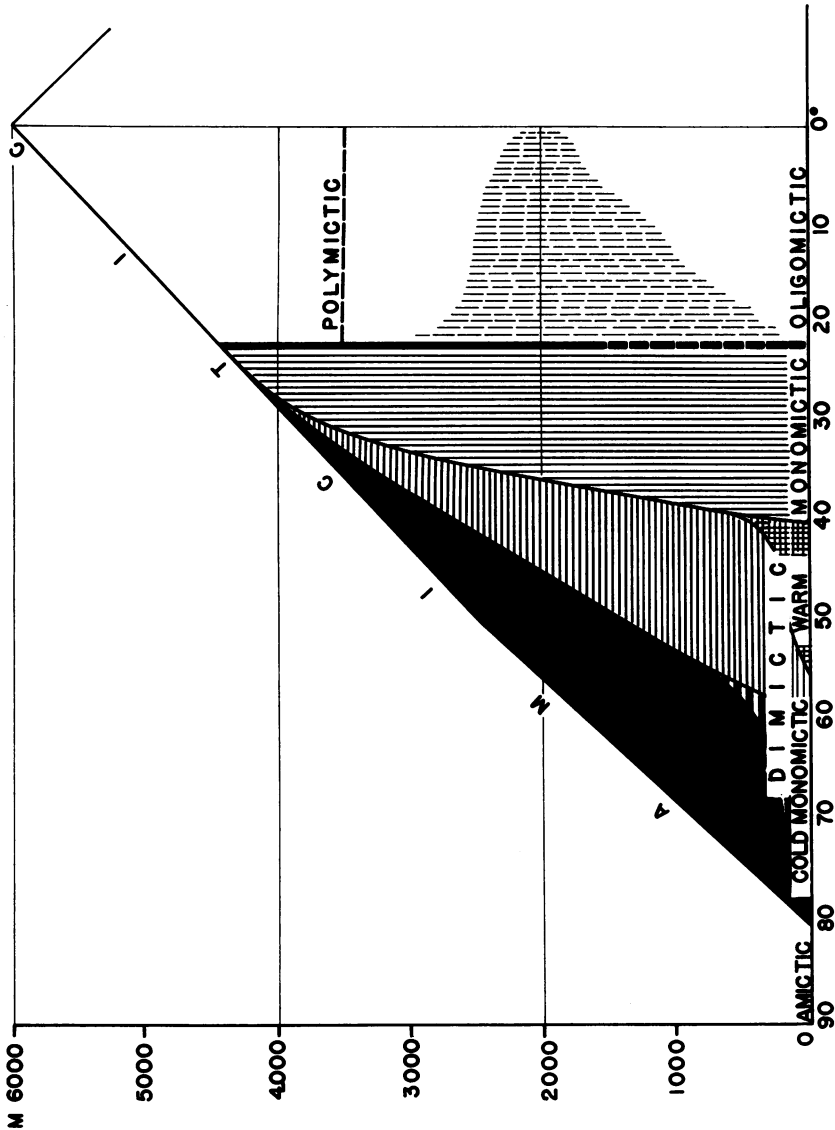


FIG. 1.—Schematic arrangement of the thermal lake types. Solid black: cold monomictic. Black and white, horizontal bars: transitional regions. Vertical lines: dimictic. Crossed lines: transitional regions. Horizontal lines: warm monomictic. The two equatorial types occupy the unshaded areas labeled oligomictic and polymictic, separated by a region of mixed types, mainly variants of the warm monomictic type, indicated by broken vertical lines. Horizontal scale in degrees; vertical scale in meters.

equatorial regions of that continent at fair elevations a great variety of conditions occur, linking the oligomictic, with a strong tendency to meromixis, as in Lake Nyasa,<sup>5</sup> the warm monomictic, as in Lake Mohasi, Ruanda,<sup>6</sup> and the polymictic, as in Lake Tana,<sup>7</sup> Lake Rudolf, Lake Albert, and, to a large extent, Lake Victoria and Lake Naivasha.<sup>8</sup> The existence of special climatic regimes, notably the

monsoonal regime of India and Ceylon, probably produces considerable disturbance of the scheme; Dr. S. Dillon Ripley, for instance, tells us that in Ceylon artificial lakes at an altitude slightly over 2,000 meters may occasionally freeze, though such lakes are certainly not polymictic and presumably belong in the dimictic category.

Further details will be found in our forthcoming independent works.

\* This paper is dedicated to Professor Alexander Petrunkevitch on his eightieth birthday. Contribution from the Osborn Zoological Laboratory, Yale University, New Haven, Connecticut.

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<sup>2</sup> F. Ruttner, "Hydrographische und hydrochemische Beobachtungen auf Java, Sumatra und Bali," in *Tropische Binnengewässer* (*Arch. Hydrobiol., Suppl.*, Vol. 8 [1931]), pp. 197–454.

<sup>3</sup> G. E. Hutchinson, *The Study of Lakes*, Vol. 1: *Geographical and Physiochemical Limnology* (to appear late in 1956).

<sup>4</sup> J. Murray, in E. H. Shackleton, *The Heart of the Antarctic* (London: W. Hienemann, 1909), Vol. 2.

<sup>5</sup> R. S. A. Beauchamp, "Hydrological Data from Lake Nyasa," *J. Ecol.*, **41**, No. 2, 226–239, 1953.

<sup>6</sup> H. Damas, "Recherches limnologiques dans quelques lacs du Ruanda," *Verhandl. Int. Ver. Limnol.*, **12**, 335–341, 1955.

<sup>7</sup> G. Morandini, *Missione di studio al Lago Tana*, Vol. 3: *Ricerche limnologiche*, Part 1, "Geografia-fisica" (Rome, 1940). This author completes the Forellian scheme by the addition of an *equatorial* type for the freely circulating lakes of the tropics. It is, however, apparent that two extreme types occur in equatorial latitudes, depending on the altitude, as is indicated in the present note.

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## GAMMA FACTORS IN FUNCTIONAL EQUATIONS

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In a so-called "functional" equation for a zeta function pertaining to an algebraic field or a modular setup, the Dirichlet series occurring is multiplied by a function  $\Delta(s)$  of the complex variable  $s = \sigma + i\tau$  which introduces itself each time by some (multiple Euler) integral of the form

$$\frac{\Delta(s) \lambda(x)}{\mu(x)^s} = \int_P e^{-(x, t)} R(t)^s d\Omega(t), \quad (1)$$

which each time, by an appropriate computation, turns out to be a product

$$\prod_{m=1}^N \Gamma(p_m s + q_m),$$

in which  $p_m$  are positive rational numbers and  $q_m$  are complex numbers.<sup>1</sup> After replacing  $\Delta(s)$  by  $\Delta(rs)$  for a suitable positive integer  $r$ , the numbers  $p_m$  may be assumed to be positive integers themselves, and, applying, now, the classical